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## DEVELOPMENT AND VERIFICATION OF AN IMPROVED THERMO DYNAMICAL MODEL FOR SINGLE SPOOL JET ENGINES<sup>34</sup>

*The main goal of the present research is to develop and verify an improved mathematical model for the thermo dynamical analyses of single spool jet engines with and without afterburner at start position. The specific points of the thermo-dynamical cycles and the main characteristics of the engines are determined by a concentrated parameter-distribution type method, which is implemented in MATLAB environment. The governing equations are based on the mass, the energy balance and the real thermo dynamical processes, in which the mechanical, isentropic and burning efficiencies, pressure losses and power reduction rate of the auxiliary systems are considered. Nonlinear constraint optimization is used for parameter sensitivity analyses and to fit the unknown parameters for the available data as thrust and thrust specific fuel consumption. Material properties, ambient conditions, incoming air mass flow rate, pressure ratio of the compressor, turbine inlet total temperature and the length and diameter of the engines are imposed as input parameters for the analyses. The temperature and component mass fraction dependent gas properties are determined by iteration cycles. Among the results of the analyses, the thrust and thrust specific fuel consumption are compared to available operational data of the БД-7, КР7-300 (w/o afterburner) and РД-9Б (w/ afterburner) engines. The corner points of the thermo-dynamical cycles are plotted in T-s diagrams for the БД-7 and РД-9Б engines.*

### EGYFORGÓRÉSSES SUGÁRHAJTÓMŰ TERMODINAMIKAI MODELLJÉNEK TOVÁBBFEJLESZTÉSE ÉS VERIFIKÁCIÓJA

*A jelen kutatás célja egy olyan matematikai modell kidolgozása és a számítási eredmények verifikálása, amely az egyszerűsített közelítéshez képest pontosabb eredményt szolgáltat az egytengelyes sugárhajtóművek termodinamikai viselkedésének leírására utánégetéses és utánégetés nélküli esetekben. A számításokat starthelyzetben végeztük el. A sűrűlódásos áramlás figyelembevétele érdekében egy koncentrált paraméter-eloszlású számítási eljárást dolgoztunk ki. A módszert MATLAB környezetben implementáltuk. A tömeg- és energia-megmaradás egyenletei mellett a termodinamikai folyamatok alapegyenletei a rendszer mechanikai hatásfokát, a kompresszor és a turbina izentropikus hatásfokát, a tüzelési hatásfokot, a nyomásvesztéségi tényezőket és a kiegészítő berendezések teljesítményfelvétel-hányadát is figyelembe vesszük. Nemlineáris korlátos optimalizációs eljárást alkalmaztunk az ismeretlen paraméterek tolóerőre és a tolóerőre vonatkoztatott fajlagos tüzelőanyag-fogyasztásra hangolása érdekében. A folyamatokban résztvevő anyagok tulajdonságai és a szükséges geometriai méretek mellett a kompresszor torlóponyi nyomásviszonyát, a turbina belépő torlóponyi hőmérsékletét, a hajtóműbe belépő levegő tömegáramát és a hajtómű kilépő keresztmetszetének átmérőjét írjuk elő bemeneti peremfeltételként a rendelkezésre álló szakirodalmi adatok alapján. Az állandó nyomáson vett fajhő és az*

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adiabatikus kitévő meghatározása esetén iteráció segítségével vettük figyelembe az egyes komponensek tömegarányát és a közeg hőmérsékletét. Az analízis során kialakult fajlagos tolóerőt és a tolóerőre vonatkoztatott fajlagos tüzelőanyag-fogyasztást hasonlítottuk össze a rendelkezésre álló adatokkal a BД-7 és KP7-300 típusú utánégetés nélküli, illetve PД-9B típusú utánégetéses hajtóművek esetén. A számítás eredményeként kialakult termodinamikai körfolyamatot T-s diagramban ábrázoltuk BД-7 és PД-9B hajtóművek esetén.

## NOMENCLATURE

$A_9$	Outlet area of the engine [m <sup>2</sup> ]
$C_f$	Specific heat of the fuel (combustion chamber) [J/kg/K]
$C_{f,A}$	Specific heat of the fuel (afterburner) [J/kg/K]
$C_{p,mix}$	Specific heat of air and gas mixture at const. pressure, at given T temperature and fuel to air ratio [J/kg/K]
$\bar{C}_{p,mix}$	Mean (between $T_i$ and $T_{i+1}$ ) spec. heat of air and gas mixture at const. pressure and at given fuel to air ratio [J/kg/K]
$D$	Diameter of the engine [m]
$f$	Fuel to air ratio in the combustion chamber [-]
$f_A$	Fuel to air ratio in the afterburner [-]
$L$	Length of the engine [m]
$L_0$	Theoretical air mass required to burn 1 kg fuel at stoichiometry
$\dot{m}_{air}$	Incoming air mass flow rate of engine [kg/s]
$\dot{m}_{air,st}$	Air mass flow rate at stoichiometry in comb. chamber. [kg/s]
$\dot{m}_{air,st,A}$	Air mass flow rate at stoichiometry in afterburner [kg/s]
$\dot{m}_f$	Mass flow rate of fuel in the combustion chamber [kg/s]
$\dot{m}_{f,A}$	Mass flow rate of fuel in the afterburner [kg/s]
$\dot{m}_{tech}$	Bleed air mass flow rate due to the technological reason [kg/s]
$\dot{m}_{4-5}$	Mass flow rate in the turbine [kg/s]
$\dot{m}_9$	Mass flow rate at the exhaust of the engine [kg/s]
$M$	Mach number [-]
$p$	Pressure [Pa]
$p_0$	Ambient static pressure [Pa]
$p_9$	Static pressure at the exhaust of the engine [Pa]
$p_c$	Critical static pressure [Pa]
$Q_R$	Lower heating value of the fuel [MJ/kg]
$Q_{RA}$	Lower heating value of the fuel in the afterburner [MJ/kg]
$R$	Specific gas constant [J/kg/K]
$r_{cc}$	Total pressure loss coefficient in the combustion chamber [-]
$r_d$	Total pressure loss coefficient in the intake duct (diffuser) [-]
$r_{td}$	Total pressure loss coefficient in the afterburner liner [-]
$T$	Temperature [K], thrust [kN]



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TSFC	Thrust Specific Fuel Consumption [kg/(kN h)]
$T_0$	Ambient temperature [K]
$T_{of}$	Temperature of the fuel at the combustion chamber [K]
$T_{of,A}$	Temperature of the fuel at the afterburner [K]
$V_{el0}$	Flight speed [m/s]
$V_{el9}$	Velocity at the exit of the exhaust nozzle [m/s]
$\gamma_{mix}$	Ratio of specific heats at given temperature for gas mixture [-]
$\bar{\gamma}_{mix}$	Ratio of mean specific heats for gas mixture [-]
$\delta_{tech}$	Bleed air ratio for technological reasons [-]
$\delta_{blade\_cooling}$	Air income ratio due to the turbine blade cooling [-]
$\zeta$	Power reduction rate for the auxiliary systems [-]
$\eta_b$	Burning efficiency [-]
$\eta_C$	Isentropic efficiency of the compressor [-]
$\eta_T$	Isentropic efficiency of the turbine [-]
$\eta_m$	Mechanical efficiency [-]
$\eta_n$	Nozzle isentropic efficiency [-]
$\pi_C$	Total pressure ratio of the compressor [-]

## INTRODUCTION

Nowadays, the most of the leading technologies are established in the aeronautical sector, wide spectrum of research and developments are in progress in that areas [10][11], and the innovation-transfer into the other industrial parties are clearly observed. This is especially true for the propulsion systems of the aircrafts. Beside the aeronautical applications, the gas turbines are generally used as energy sources in the other contributions of transportation and the energy, oil and gas sectors of the industry. These types of engines have high power-density ratio ( $\sim 20\text{--}30$  HP/kg) compared to piston engines ( $\sim 1\text{--}2$  HP/kg) [9]. The gas turbines are relatively light-weight structures and have compact sizes, which makes their installation cost efficient. These engines are less sensitive for the overloads, they have less solid cross sectional area against the flow and have less vibration due to the well balanceable and rather axisymmetric rotating components. The gas turbines have high availability (97%) and reliability ( $> 99\%$ ), they have low emission (there is no lubricant in the combustion chamber and no soot during transient loads) they contain less moving parts and represent less sensitivity for the quality of the fuel compared to the piston engines. Additionally, there is no need for liquid-based cooling system, but the maximum allowable temperature ( $\sim 1500$  °C) at the turbine inlet section must be limited due to the metallurgical reasons [9].

Beside the technical characteristics of the gas turbines today, certain amounts of potentials are available for improving their efficiencies, performances and emissions over the wider range of operational conditions [12][13]. Although the experiences and the know-how of the gas turbine manufacturers increasing continuously, the different mathematical models with using of optimum choice and form of



the most dominant processes can significantly contribute to decrease the cost, time and capacity in the early phase of gas turbine design and developments. There are many scientific publications are subjected to the thermodynamic-based simulation approaches, which confirms also the need for creating more and more accurate calculation methods. Rahnan and others [1] presented analytical cycle analyses for gas turbine engine with simple and regenerative operational mode with parameter sensitivity analyses. Silva and his co-workers [2] published an evolutionary approach as the optimization framework to design for optimal performance in terms of minimizing fuel consumption while maintaining nominal thrust output, maximizing thrust for the same fuel consumption and minimizing turbine blade temperature. Yarlagaadda [3] focused on performance analysis of the J85 turbojet engine with an inlet flow control mechanism to increase RPM (and pressure ratio) for higher efficiency and the same thrust values by using 1-D non-linear unsteady equations.

In the present paper, the thermo-dynamical cycles of single spool turbojet engines are analysed by developing and applying a concentrated parameter distribution-type method. The computational procedure used in the analytical model relies on the mass and energy balance with frictional and polytropic thermo dynamical processes. The constrained nonlinear optimisation is used for parameter sensitivity analyses and for determining the unknown parameters as efficiencies, losses, power reduction rates of the auxiliary systems, bleed air ratios for technological reasons, air income ratios due to blade cooling and total temperature at the afterburner (in case of need) by means of having parameter-state, which provides the closest results to the available thrust and thrust and thrust specific fuel consumption of single spool turbojet engines in start condition. The temperature and component mass fraction dependent gas properties as specific heat at constant pressure and ratio of specific heats are determined by iteration cycles. The results of the simulations are compared to the available operational data, which are found in [6].

## The Novell Computational model

### General Aspects

Commonly valid items are discussed in the present part of the model description.

ISA (International Standard Atmosphere) is applied for determining ambient parameters as pressure and temperature at static sea level condition in this case:

- Ambient pressure:  $p = p_0 = 101325 \text{ Pa}$
- Ambient temperature:  $T = T_0 = 288 \text{ K}$
- Mach number:  $M = 0$

There are initially given parameters in the available specification of the investigated engines [6] as incoming air mass flow rate, pressure ratio of the compressor, turbine inlet total temperature and the length and the diameter of the engine, which are imposed as input parameters of the analyses, meanwhile there are many others as efficiencies (mechanical, isentropic of compressor and turbine, burning and exhaust nozzle), losses (pressure loss of inlet diffuser, combustion chamber and

afterburner or turbine exhaust pipe), power reduction rates of the auxiliary systems, bleed air ratios for technological reasons, air income ratios due to blade cooling and total temperature at the afterburner (if it is the case), which are not available. The strategy of the present work is to use the available parameters as inputs together with material properties, ambient pressure and temperature, meanwhile the unknown parameters are identified by constrained optimization to recover the thrust and thrust specific fuel consumption, which are also given in the specification [6].

Beside the constant material properties as specific gas constant, the specific heat at constant pressure and the ratio of the specific heats at constant pressure and volume are determined by considering the temperature (T) and fuel to air ratio (f or  $f_A$ ). These parameters are changed at each cross sections of the engine depends on the local thermal and mass fraction conditions. (1)–(3) shows the expressions how they are determined depends on the mean value through the certain process (2, 3) or stand alone value (real specific heat) at given temperature and fuel mass fraction in the certain cross section of the engine (2, 3). Iteration processes are used if the temperatures or mass fractions are the functions of the unknown parameters in order to find the equilibrium between the temperature dependent material properties and thermo dynamical variable.

$$\bar{C}_{p\text{mix}}(T_i, T_{i+1}, f \text{ or } f_A) = \frac{1000 \sum_{j=0}^n \frac{a_j + f \text{ or } f_A c_j}{(f \text{ or } f_A + 1)} \left[ \left( \frac{T_{i+1}}{1000} \right)^{j+1} - \left( \frac{T_i}{1000} \right)^{j+1} \right]}{T_{i+1} - T_i} \quad (1)$$

$$C_{p\text{mix}}(T, f \text{ or } f_A) = \sum_{j=0}^n \frac{a_j + f \text{ or } f_A c_j}{f \text{ or } f_A + 1} \left( \frac{T}{1000} \right)^j \quad (2)$$

$$\bar{\gamma}_{\text{mix}} = \frac{\bar{C}_{p\text{mix}}(T_i, T_{i+1}, f \text{ or } f_A)}{\bar{C}_{p\text{mix}}(T_i, T_{i+1}, f \text{ or } f_A) - R} \quad \text{or} \quad \gamma_{\text{mix}} = \frac{C_{p\text{mix}}(T, f \text{ or } f_A)}{C_{p\text{mix}}(T, f \text{ or } f_A) - R} \quad (3)$$

The  $a_j$  and  $c_j$  are the polynomial constants for air and kerosene fuel according to [4]. Table 1 contains the values of the polynomial constants of gases.

$a_j$	Value	$c_j$	Value
$a_0$	1043.797	$c_0$	614.786
$a_1$	-330.6087	$c_1$	6787.993
$a_2$	666.7593	$c_2$	-10128.91
$a_3$	233.4525	$c_3$	9375.566
$a_4$	-1055.395	$c_4$	-4010.937
$a_5$	819.7499	$c_5$	257.6096
$a_6$	-270.54	$c_6$	310.53
$a_7$	33.60668	$c_7$	-67.426468

Table 1. Polynomial constants used for computing the material properties of gases [4]

## Thermo-dynamical model of the single-spool turbojet engine

A meridian cross section of a typical single spool turbojet engine with afterburner is shown in Figure 1. The ambient air from stage 0 enters into the engine at cross section 1 and passes through the inlet diffuser till the cross section 2. Then the flow enters into the compressor. Here, between cross sections 2–3, the air is compressed. The next part of the engine is the combustion chamber, where the fuel is injected and the heat-generation is realised between the stages 3–4. The products of the combustion are cooled down by the secondary flows before entering into the turbine across the section 4. The gas is expanded within the turbine, which provides power for the compressor. The gas stream leaves the turbine across the stage 5 and, if it is the case, goes through the afterburner, at which significant amount of heat is generated also to increase the exhaust speed and so the thrust. Then the expanded gases enter into the exhaust nozzle across the section 7 and producing thrust after leaving the engine at stage 9.

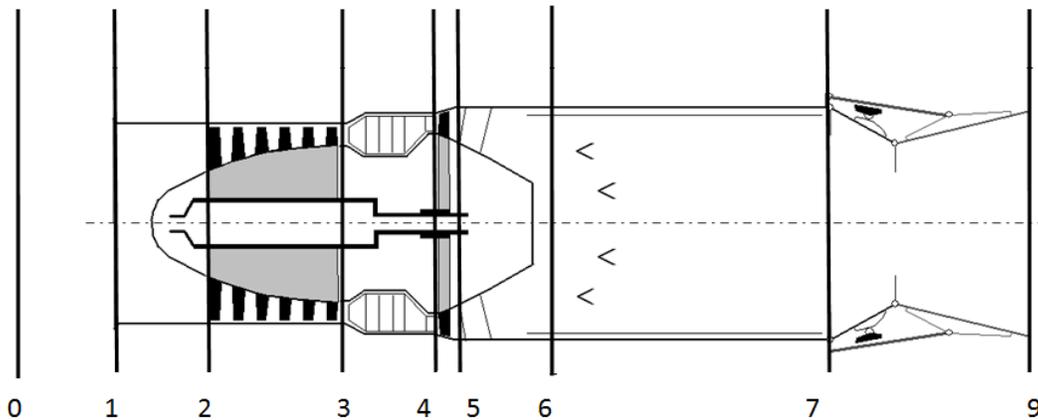


Figure 1. Layout of the single spool turbojet engine with afterburner [8]

Regarding to most general aspects, the first step of the analysis is to determine the total temperature and pressure at the intake, by using equations (4, 5 and 6) [7].

$$T_{02} = T_{01} = T_0 \left( 1 + \frac{\gamma-1}{2} M^2 \right) \quad (4)$$

$$p_{01} = p_0 \left( 1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}} \quad (5)$$

The total pressure at compressor intake is determined by the total pressure recovery factor of the intake:

$$p_{02} = p_{01} r_d \quad (6)$$

The total pressure at outlet of the compressor is calculated by the given total pressure ratio of the compressor as follows:

$$p_{03} = \pi_c p_{02} \quad (7)$$



The total temperature at the outlet of compressor is determined by using the isentropic efficiency of the compressor (8). Iteration cycle is used at this stage due to the fact that the specific heat at constant pressure is also depends on the total temperature at this process.

$$T_{03} = T_{02} + \frac{\bar{c}_{p\text{mix}}(T_{02}, T_{03s}, f)(T_{03s} - T_{02})}{\bar{c}_{p\text{mix}}(T_{02}, T_{03}, f)\eta_c} \quad (8)$$

The mass flow rate of the fuel can be computed by the total enthalpy balance of the combustion chamber (9) by taking into account that the maximum turbine inlet total temperature is given. Equation (9) contains five terms as 1: the stagnation enthalpy of the incoming pure air into the combustion chamber, 2: the stagnation enthalpy of the fuel, 3: heat generation by the combustion, 4: stagnation enthalpy of the hot gas at the stoichiometric burning, which leaves the combustion chamber, 5: stagnation enthalpy of the pure air, which is available at the outlet section of the combustion chamber.

$$\underbrace{\dot{m}_{\text{air}}(1 - \delta_{\text{tech}})C_{p\text{mix}}(T_{03}, f = 0)T_{03}}_1 + \underbrace{\dot{m}_f C_f T_{0f}}_2 + \underbrace{\eta_b Q_R \dot{m}_f}_3 = \underbrace{(\dot{m}_{\text{air},st} + \dot{m}_f)C_{p\text{mix}}(T_{04}, f)T_{04}}_4 + \underbrace{(\dot{m}_{\text{air}}(1 - \delta_{\text{tech}}) - \dot{m}_{\text{air},st})C_{p\text{mix}}(T_{04}, f)T_{04}}_5 \quad (9)$$

Behind the compressor, the mass flow rate of the air removal is considered due to the technological reason ( $\delta_{\text{tech}} = \dot{m}_{\text{tech}} / \dot{m}_{\text{air}}$ ). Kerosene with lower heating value  $Q_R = 42 \text{ MJ/kg}$  is used.  $\dot{m}_{\text{air},st}$  is the mass flow rate of the air, which involved in the burning at stoichiometric condition, and which is determined by equation (10) ( $L_0 = 14.72 \text{ kg/kg}$ ).

$$\dot{m}_{\text{air},st} = \dot{m}_f L_0 \quad (10)$$

$$f = \frac{\dot{m}_f}{\dot{m}_{\text{air}}(1 - \delta_{\text{tech}})} \quad (11)$$

Iteration cycle is used at that section also to consider the variation of fuel to air ratio in the specific heat at constant pressure.

After applying the energy balance in the combustion chamber, the power equilibrium of the compressor and the turbine is applied to determine the total temperature at the outlet section of the turbine ( $T_{05}$ ):

$$\dot{m}_{\text{air}} \bar{c}_{p\text{mix}}(T_{02}, T_{03}, f = 0)(T_{03} - T_{02}) = \eta_m \dot{m}_{4-5} (1 - \zeta) \bar{c}_{p\text{mix}}(T_{04}, T_{05}, f)(T_{04} - T_{05}) \quad (12)$$

The  $\dot{m}_{4-5}$  is the mass flow rate in the turbine ( $\dot{m}_{4-5} = \dot{m}_{\text{air}}(1 - \delta_{\text{tech}})(1 + f)(1 + \delta_{\text{blade\_cooling}})$ ), in which the cooling mass flow rate of air is considered to prevent the turbine blades from the thermal loading of the high temperature gases. The cooling air is generally a part of the bleed air extracted from the compressed air close to the outlet section of the compressor.



The engine section 5-6 (see Figure 1.) is not considered (or distinguished) in the present case. It should be mentioned that the pressure loss develops at that region is included in the next section as 6-7, which approximation has a negligible effect for the output of the analyses.

If it is the case, the afterburner locates between the cross sections of 6 and 7 (see Figure 1). Total enthalpy balance has been applied for this segment in order to determine the fuel mass flow rate enters into afterburner ( $\dot{m}_{f,A}$ ). The maximum allowable temperature ( $T_{07}$ ) is obtained by a constrained nonlinear optimization procedure together with other unknown parameters as losses and efficiencies for example. The left hand side of equation (13) represents the incoming energy to system and the right hand side of it shows the leaving ones. The first term of equation (13) is the stagnation enthalpy of pure air, which is available at section 6, and,  $\dot{m}_{6,air} = \dot{m}_{5,air} = \dot{m}_{air}(1 - \delta_{tech})(1 + \delta_{blade\_cooling})$ . The second and third term is the total enthalpy of the fuel entering into afterburner and heat generated by the combustion respectively. The last term in the left hand side is the total enthalpy of the incoming hot gases, which are already burnt previously in the combustion chamber. The first two terms in the right hand side of equation (13) represents the total enthalpy of the hot gases of the combustion chamber and the afterburner, meanwhile the last term is the total enthalpy of the pure air, which leaves the system.

$$(\dot{m}_{6,air} - \dot{m}_{air,st}) C_{p,mix}(T_{06}, f = 0)T_{06} + \dot{m}_{f,A} C_{f,A} T_{0f,A} + \eta_{b,A} Q_{R,A} \dot{m}_{f,A} + (\dot{m}_{air,st} + \dot{m}_f) C_{p,mix}(T_{06}, f)T_{06} = (\dot{m}_{air,st} + \dot{m}_f) C_{p,mix}(T_{07}, f)T_{07} + (\dot{m}_{air,st,A} + \dot{m}_{f,A}) C_{p,mix}(T_{07}, f_A)T_{07} + (\dot{m}_6 - \dot{m}_{air,st} - \dot{m}_{air,st,A}) C_{p,air}(T_{07}, f = 0)T_{07} \quad (13)$$

Iteration cycle is used at that section also to consider the dependences of the afterburner fuel to air ratio in the specific heat at constant pressure ( $f_A = \dot{m}_{f,A}/\dot{m}_{6,air}$ ).

The total temperatures at the inlet section of the nozzle will be equal with the total temperature at the exit of the turbine in case of engines without afterburner.

The total pressure at the inlet section of the exhaust nozzle is determined by the function of the turbine outlet total pressure and the total pressure loss coefficient of the afterburner liner or turbine exhaust pipe:

$$p_{07} = p_{05} \Gamma_{td} \quad (14)$$

Expansion process develops in the exhaust nozzle, which is shown in Figure 2. By having the inlet conditions of the nozzle ( $p_{07}, T_{07}$ ) and its isentropic efficiency, it can be determine whether the converging nozzle is choked or unchoked one. After having this information, the pressure, the temperature and the velocity at the exit of the exhaust nozzle can be calculated. If the ambient pressure is higher than the critical pressure, than the nozzle flow is unchoked one, therefore the exhaust pressure of the exit is equal to the ambient pressure. At other case, at which the critical pressure is higher than the ambient pressure, the nozzle flow is choked, and then the exhaust pressure of the nozzle is equal to the critical pressure  $p_c$ .

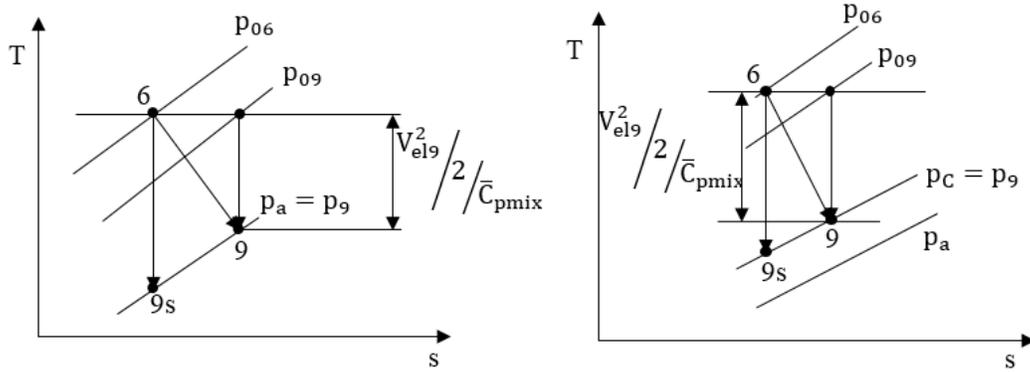


Figure 2. Nozzle flow with losses for unchoked (left hand side) and for choked (right hand side) flow conditions

New analytical equation has been derived for determining the critical pressure at the exhaust of the nozzle, in which beside the dependences of temperature variation and fuel to air ratio in the specific heat at constant pressure, the ratio of the specific heats is also considered. The critical static pressure at the outlet section of the exhaust system is coupled with the total and static temperatures, hence iteration cycles are used to determine the corresponding critical pressure and belonging total and static temperatures, which satisfy equation (15) and provides thermo dynamical parameters in case of choked conditions. The fuel to air ratio of the afterburner is also included in  $f$  in case of engine with afterburner:  $f = (\dot{m}_f + \dot{m}_{fA}) / (\dot{m}_{6,air} - \dot{m}_{air,st} - \dot{m}_{air,st,A})$ .

$$p_c = p_{07} \left[ 1 - \frac{1}{\eta_n} \left( 1 - \frac{2\bar{c}_{pmix}(T_{09}, T_9, f)}{2\bar{c}_{pmix}(T_{09}, T_9, f) + \gamma_{mix}R} \right) \frac{\bar{c}_{pmix}(T_{09}, T_9, f)}{\bar{c}_{pmix}(T_{09}, T_{9s}, f)} \right]^{\frac{\bar{\gamma}_{mix}}{\bar{\gamma}_{mix} - 1}} \quad (15)$$

The ratio of specific heats (16) and (17) are used in equation (15).

$$\bar{\gamma}_{mix} = \frac{\bar{c}_{pmix}(T_{09}, T_{9s}, f)}{\bar{c}_{pmix}(T_{09}, T_{9s}, f) - R} \quad (16)$$

$$\gamma_{mix} = \frac{c_{pmix}(T_9, f)}{c_{pmix}(T_9, f) - R} \quad (17)$$

## Output parameters of the simulations

The thrust and thrust specific fuel consumption are the two output parameters of the analyses, which are available in the open literature, they are the goal functions of the constrained optimisation in case of parameter fitting, and so they can be used for the model verification.

The thrust can be expressed as follows:

$$T = \left[ (\dot{m}_{air}(1 - \delta_{tech})(1 + \delta_{blade\_cooling})(1 + f + f_A)) \right] V_{el9} - \dot{m}_{air} V_{el0} + A_9(p_9 - p_0) \quad (18)$$

The thrust specific fuel consumption is defined as the total mass flow rate of the fuel (including the mass flow rate of the afterburner, in case of engines with afterburner) divided by the thrust (19).

$$\text{TSFC} = \frac{\dot{m}_f + \dot{m}_{fA}}{T} \quad (19)$$

## RESULTS AND DISCUSSION

Three gas turbines have been selected for the analyses and for the verification of the simulation method as BД-7, PД-9Б and KP7-300 turbojet engines. The available specifications of these gas turbines are found in [6]. The input data of the simulation are the incoming air mass flow rate to the engine, pressure ratio of the compressor, turbine inlet total temperature and the length and the diameter of the engine beside with material properties, ambient pressure and temperature (see Table 2). Unfortunately, there are many other parameters as efficiencies (mechanical, isentropic of compressor and turbine, burning and exhaust nozzle), losses (pressure loss of inlet diffuser, combustion chamber and afterburner or turbine exhaust pipe), power reduction rates of the auxiliary systems, bleed air ratios for technological reasons, air income ratios due to blade cooling and total temperature of the afterburner (if it is the case), which are not available. Hence, constrained nonlinear optimisation is used for determining these unknowns to find such a parameter-state, which provides the closest results to the available thrust and thrust specific fuel consumption for the engines. The results of the optimization are found in Table 3. The parameters are in the reasonable range, so they can be accepted.

Type of turbojet engines	Input data, which are used in the simulations (start positions)				Available data (start positions)	
	$T_{04}$ (K)	$\pi_K^{to}$	$\dot{m}_{air}$ (kg.s <sup>-1</sup> )	$\frac{L}{D}$ (m/m)	T (kN)	TSFC (Kg. kN <sup>-1</sup> h <sup>-1</sup> )
single spool engine (BД-7)	1090	11.2	187	4.25/1.29	107.8	82
single spool engine (PД-9Б) with afterburner	1150	7.5	43.3	5.56/1.66	32.4	163
single spool engine (KP7-300)	1330	4.5	35.5	2.01/1.64	21.1	132

Table 2. Operational data of the BД-7, PД-9Б and KP7-300 turbojet engines [6]

Type of turbojet engines	Efficiencies, losses power reduction rates of the auxiliary systems, bleed air ratios for technological reasons, air income ratio due to blade cooling and total temperature at the afterburner											
	$r_d$	$r_{cc}$	$r_{td}$	$\eta_{izc}$	$\eta_{izT1}$	$\eta_{m1}$	$\eta_b$	$\eta_n$	$\xi$	$\delta_{tech}$	$\delta_{bc}$	$T_{07}$ (K)
single spool engine (BД-7)	.96	.94	.98	.85	.85	.98	.98	.98	.009	.15	.15	-
single spool engine with afterburner (PД-9Б)	.95	.95	.95	.85	.89	.98	.98	.96	.005	.15	.15	1870
single spool engine (KP7-300)	.94	.92	.98	.85	.85	.98	.99	.98	.01	.18	.12	-

Table 3. Identified parameters of the 3 investigated turbojet engines

Considering the available data, only the thrust (T.) and thrust specific fuel consumption (TSFC) are used directly to check how far the calculated results of the analyses from the data available in



the specifications. These data are presented in Table 4 and show remarkable agreement with each other. Although the relative deviations between the computed and available parameters are less than 2.8 % and so the accuracy of the developed concentrated parameter-distribution type method is acceptable, further investigation - as the extension of the model for dual spool and bypass engines - with additional verifications and validations are needed.

Type of turbojet engines	Available data for the verification (start position)		Results of the simulations (start position)		Relative errors	
	T (kN)	TSFC(kg. kN <sup>-1</sup> h <sup>-1</sup> )	T(kN)	TSFC(Kg. kN <sup>-1</sup> h <sup>-1</sup> )	T	TSFC
single spool engine (ВД-7)	107.8	82	109	81.7	0.74%	0.3%
single spool engine (РД-9Б) with afterburner	32.4	163	32.4	162.94	0 %	0.03%
single spool engine (КР7-300)	21.1	132	21.7	132.7	2.8%	0.5%

Table 4. Comparisons of available data with the results of simulations

The real thermo dynamical cycles of the ВД-7 turbojet engine without afterburner and РД-9Б turbojet engine with afterburner are calculated and plotted in T-s diagrams (see Figures 3. and 4. respectively).

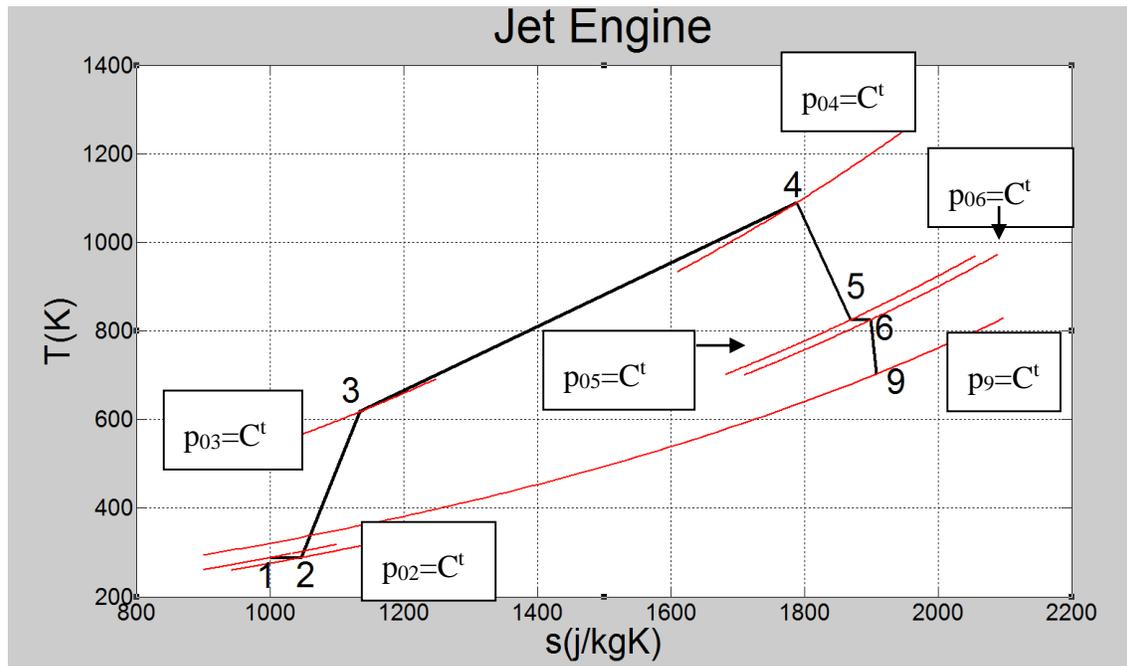


Figure 3. T-s diagram of the ВД-7 turbojet engine without afterburner

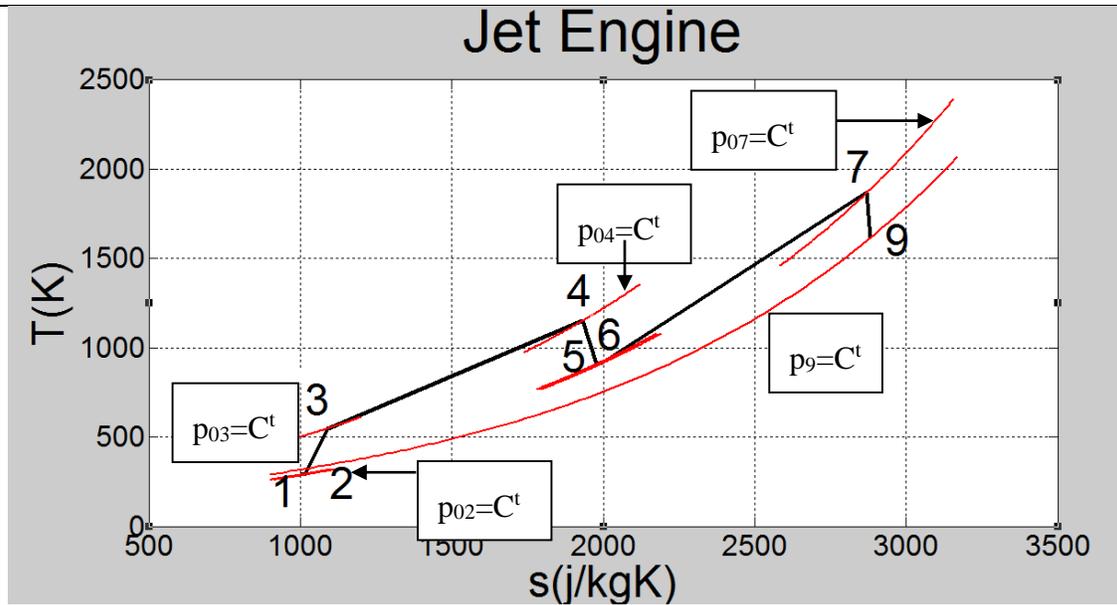


Figure 4. T-s diagram of the PD-9B turbojet engine with afterburner

## CONCLUSIONS

An improved mathematical model has been developed for the thermo dynamical analysis of the single spool jet engines with and without afterburner at start position. The specific points of the thermo-dynamical cycles and the main characteristics of the engines are determined by a concentrated parameter-distribution type method, which is implemented in MATLAB environment. The governing equations are based on the mass, the energy balance and the real thermo dynamical processes, in which the mechanical, isentropic and burning efficiencies, pressure losses and power reduction rate of the auxiliary systems are considered. Nonlinear constraint optimization is used for parameter sensitivity analyses and to fit the unknown parameters for the available data as thrust and thrust specific fuel consumption. Material properties, ambient conditions, incoming air mass flow rate, pressure ratio of the compressor, turbine inlet total temperature and the length and diameter of the engine are imposed as input parameters of the analyses. Concerning the material properties, the temperature and component mass fraction dependent gas properties are determined by iteration cycles. Amongst the results of the analyses, the specific thrust and thrust specific fuel consumption are compared to available operational data of the BД-7 and KP7-300 (w/o afterburner) and PD-9B (w/ afterburner) engines. The corner points of the thermo-dynamical cycles are plotted in T-s diagrams for the BД-7 and PD-9B engines. The verification of the simulation shows that the available and calculated thrusts and thrust specific fuel consumptions are aligned with each other and the deviation between them is less than 2.8 %. Hence, although further verifications and validations are needed, it can be concluded that the accuracy of the developed concentrated parameter-distribution type model is acceptable in engineering point of view.



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