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Contents

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Contents

An MRP-based integer programming model for capacity planning

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Abstract

Capacity planning decisions represent an important topic in the operations management literature, as they greatly influence the performances of the firm and determine if the firm is able to satisfy market demand in each period. In order to be able to deal with uncertainties and fluctuations of market demand and other elements of the internal and external environment, capacity planning decisions are broken down into several levels of aggregation. This paper builds a general integer programming model for capacity planning at the master production scheduling (MPS) level. Input data and parameters for the model can be gathered from a typical MRP/MRP II-based information system. Using the integer programming model we can determine the optimal level of production capacities, at which operating costs are minimal. The last chapter of the article presents a case study, where we use the integer programming model built to investigate what cost reductions are possible by optimizing capacity levels at a small-sized firm from the textile industry.

Keywords: capacity planning, MRP/MRP II system, production planning and control, integer programming, master production schedule.

1. Introduction

Capacity planning and control decisions are one of the most fundamental decisions in operations management as they provide the capability of a company to satisfy current and future market demand. Additionally, capacity planning decisions provide a rigid framework and constraint for other operations and production decisions. In a continuously changing, dynamic environment firms face many uncertainties, like variable customer demand, and the management of these firms has to find a way to effectively deal with these uncertainty factors. In capacity planning the main factor that influences a firm's operational efficiency is fluctuating market demand. A firm can deal with demand uncertainties basically in two different ways on aggregate level (Vörös 2007): actively attempt to change demand patterns to fit capacity availability (1), or reactively adjust production capacities to deal with the fluctuations in demand

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(2). In this paper we will investigate the second option to deal with changes in demand. The second, reactive option includes two different capacity strategies: the firm can predict future demand fluctuations and set production capacity to a constant level, which still makes possible to fulfill market demand in each period, or it can attempt to follow the fluctuation of market demand by adjusting capacity levels in each period. In this article we try to build an integer programming model which follows the latter strategy and changes production capacity levels accordingly to market demand fluctuations.

In order to achieve this, in the second chapter we will demonstrate that capacity planning decisions play a crucial role in a firm's operations strategy and can have a major impact on a firm's overall production efficiency. In the third chapter we also briefly review these important capacity decisions are broken down into multiple hierarchical levels in order to find the best capacity configuration possible. The fourth chapter investigates the connection between production planning and control (PPC) information systems and the capacity planning process, highlighting those elements of a PPC system which can offer a high quality input data for capacity planning on the master production scheduling (MPS) level. Based on these input data, in the fifth chapter, we build a general linear programming model with integer variables for capacity planning on the MPS level. The sixth chapter contains a case study which uses the model built in the previous chapter to demonstrate in a real case that by determining the optimal capacity level the firm can improve the efficiency of its production system and the firm's overall performance.

2. The strategic role of capacity planning decisions

Capacity planning is an important part of operations strategy as it plays a critical role in the success of an organization. In operations management capacity planning problems are one of the most critical ones, since all other operations planning problems are managed within the framework of the capabilities set by the capacity plan. Production capacity decisions for new, expanding, and existing facilities have a direct impact on a firm's competitive position and resulting profitability (Hammersfahr et al. 1993). There are several reasons, which make capacity decisions one of the most fundamental decisions of strategic operations management (Stevenson 1996):

- Capacity decisions have a major impact on a firm's ability to meet future demand for products and services;
- There is a significant relationship between capacity and operating costs. Costs of over- and undersized capacity should always be taken into consideration in strategic planning.

• The strategic importance of capacity decisions also lies in the initial cost involved. Required production capacity is usually a major determinant of investment costs.

•The importance of capacity decisions stems from the often required long-term commitment of resources and the fact that, once they are implemented, it may be difficult or impossible to modify those decisions without incurring major costs.

Referring to the important role of capacity planning and controlling problems, other authors argue that the decisions operations managers take in devising their capacity plans will affect several different aspects of performance (Slack et al. 2007):

• Costs will be affected by the balance between capacity and demand. Capacity levels in excess of demand could mean under-utilization of capacity and therefore high unit costs.

• Revenues will also be affected by the balance between capacity and demand, but in the opposite way. Capacity levels equal to or higher than demand at any point in time will ensure that all demand is satisfied and no revenue is lost.

• Working capital will be affected if an operation decides to build up finished goods inventory prior to demand. This might allow demand to be satisfied, but the organization will have to fund the inventory until it can be sold.

• Quality of goods or services might be affected by a capacity plan, which involved large fluctuations in capacity levels, by hiring temporary staff for example. The new staff and the disruption to the operation's routine working could increase the probability of errors being made.

• Speed of response to customer demand could be enhanced, either by the build-up of inventories (allowing customers to be satisfied directly from the inventory rather than having to wait for items to be manufactured) or by the deliberate provision of surplus capacity to avoid queuing.

• Dependability of supply will also be affected by how close demand levels are to capacity. The closer demand gets to operation's capacity ceiling, the less able it is to cope with any unexpected disruptions and the less dependable its deliveries of goods and services could be.

• Flexibility, especially volume flexibility, will be enhanced by surplus capacity. If demand and capacity are in balance, the operation will not be able to respond to any unexpected increase in demand.

3. Main objective and structure of capacity planning decisions

Capacity planning is a very complex decisional process and, as we have seen, can have a major impact on a firm's competitive position. As the most important goal of capacity planning is to meet future customer demand, running a production system with sufficiently large capacities seems to be a priority. However, having too much capacity leads to underutilization of capacities and wasted capital costs, which weaken the competitive position of a firm. On the other hand, having less capacity than required in a certain period of time leads to unfulfilled market demand, which can damage the firm's public image and incurs the costs of lost sales. Therefore, over-utilization of capacities has also a negative impact on the firm's competitive position.

Thus, capacity planning involves delicate decisions, which balance between the two extremities illustrated above. Uncertainties regarding future market demand increase the difficulty of this problem. However, finding the optimal solution of capacity-sizing problems and expanding/reducing capacities at the right moment of time can significantly contribute to the efficiency of a production system.

In order to find an optimal, balanced solution, capacity planning decisions are usually made at multiple hierarchical levels in an organization, which involve different planning horizons. Usually, we can identify two major levels of capacity decisions, which are part of either the long-term, or the short-term capacity planning process (Krajewski et al. 2007, Wortman et al. 1996). Bahl (Bahl 1991) describes managerial decisions in capacity planning as a hierarchy of decisions, where capacity planning involves long-term decisions with a planning horizon of 3-10 years, while capacity adjustments and balancing involve medium-range decisions with a planning horizon of 1-3 years. In today's rapidly changing markets these numbers may seem somewhat rough and large, when flexibility and quick responsiveness to environmental changes is considered to be a key factor to market success. Still, the hierarchisation of these decisions shows the complexity of capacity planning process, involving different planning horizons. Long- term capacity planning decisions are also referred to as rough-cut capacity check (RCCC) or simply capacity planning, while short-term capacity planning is also referred to as capacity requirements planning (CRP) or detailed capacity planning (DCP) (Gupta et al. 2006, Wortman et al. 1996). The first category represents the planning process which decides how much of each capacity resource to install, while the second category allocates available capacity among different products or jobs (Balachandran et al. 1997). Actually, RCCC is a capacity availability check applied at the master production schedule level for longer-term planning purposes, while DCP is a capacity availability check applied to planned order releases for shorter-term planning purposes (Ding et al. 2007).

Another approach to these concepts suggests that the entire planning process is started from the highest level of product aggregation and it is continued until the lowest possible level of detail is reached. The typical hierarchy of plans involves different approaches to capacity planning at every level:

Table 1. Typical hierarchy of capacity planning decisions

	Aggregation level	**Type of capacity planning**
1.	Aggregate planning	Resource Requirements Planning (RRP)
2.	Master Production Scheduling (MPS)	Rough-Cut Capacity Planning (RCCP)
3.	Material Requirements Planning (MRP)	Capacity Requirements Planning (CRP)
4.	Shop floor control	Input-output analysis

Source: Diaz - Laguna 1996

According to the same authors, subdividing the capacity planning problem into hierarchical levels according to different degrees of aggregation has the advantage of increasing the tractability of the corresponding subproblems. In addition, these subproblems usually map to different responsibility levels of the organizational structures of most companies.

This paper focuses mainly on long-term capacity planning (Rough-cut capacity planning) and on the timing of capacity expansions/reductions, but in the case-study presented we include also some short-term considerations regarding capacity adjustments into the capacity planning model. Looking at the aggregation level we focus basically on the Master Production Scheduling level by performing a rough-cut capacity check. However, by including in our model the possibility to use overtime, we reach down in the hierarchy until the Material Requirements Planning level, and identify some adjustment possibilities on the level of different product types.

4. The role of information systems in capacity planning decisions

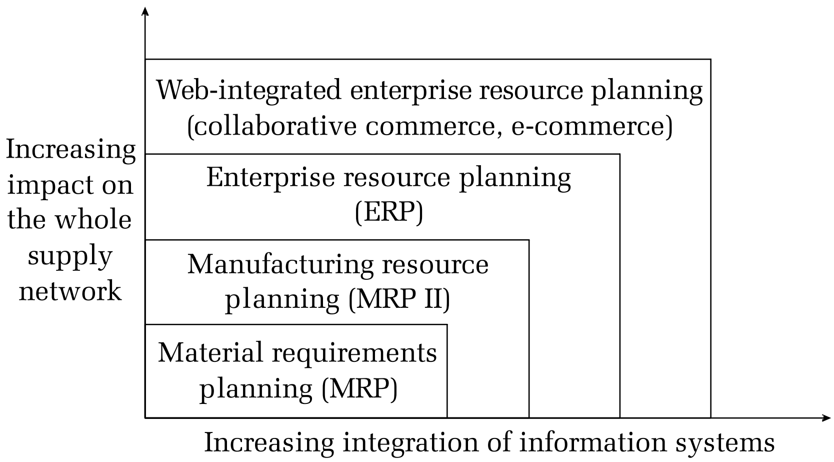
For an efficient capacity planning process there is a need for accurate information regarding (Naghi et al. 2009):

1. Quantities of expected future demand for different product types;
2. Duration and type of manufacturing processes needed (or „machine times”) in order to produce the product types demanded;

3. Current capacities available regarding every type of manufacturing processes (or number of different machine types available);
4. Manufacturing costs on process level (monthly cost of maintaining and operating different types of machines, cost of labor force, cost of overtime etc.).

These information are needed to develop a capacity plan which enables the production of different product types - in the demanded quantity and at the lowest possible cost. This requires a capacity plan, which minimizes the cost of over- and underutilization of production capacities, but does not incur the cost of lost sales.

Information systems for production planning and control usually posses a database, which contains a major part of the information, needed as input for an efficient capacity planning process. As information solutions evolved starting from Materials Requirement Planning (MRP) systems, through Manufacturing Resource Planning (MRP II) systems, reaching the Enterprise Resource Planning (ERP) system's complexity, capacity planning techniques were tried to be included as part of these systems, with greater or lesser success. Original MRP systems aimed at efficient scheduling of production requirements so that raw materials, components and subassemblies can be provided in the right amount and at the right time (Ram et al. 2006). ERP systems today are the latest and the most significant development of the original MRP philosophy. The historical development of ERP systems can be followed on figure 1.



Source: Slack et al. 2007

Figure 1. The evolution of MRP-based production planning and control systems

Many authors state (Du - Wolfe 2000, Gould 1998), that one major problem of MRP-based information systems is that they mainly ignore capacity constraints, assuming infinite resources (machine capacity and labor). Capacity Requirements Planning (CRP) and its complementary systems at different levels of aggregation were added to MRP II systems and they tried to correct this situation by identifying under-utilization and overload conditions at a machine or manufacturing unit. Some authors report that even CRP systems in an MRP II environment provide only information regarding a capacity problem, but not a viable solution to the planner (Wuttipornpun - Yenradee 2004).

In this paper we develop a simple capacity planning system based on the 4 types of information, listed at the beginning of this chapter. Looking at an MRP/MRP II based information system as a set of algorithms and objects, the needed information is provided by the following parts (units, objects, data tables) of the system:

1. Fully functional MRP II systems include a unit, which supports the *sales and marketing function* of the company. This unit next to order entry and billing is also responsible for projecting, forecasting product demand based on historical data (Duchessi et al. 1989). Using powerful statistical tools they can provide vital input information for capacity planning regarding the expected demand of different product types. Having the expected demand we can build the future master production schedule (MPS), which is a prerequisite to perform the rough-cut capacity check (Diaz - Laguna 1996).
2. Machine types and machine times needed to produce the demanded types of products is included in the *operations list*. This describes the duration and sequence of tasks to be performed to produce a finished good or a fabricated component.
3. As MRP systems evolved to MRP II systems, a *larger database* was also created which included data not only referring to the type and quantity of materials needed in production, but also regarding the number of machines, types of capacities and labor force needed (Stegerean 2002). This information provides us the actual capacity constraints, which have to be taken into consideration during the capacity planning process.
4. The evolutionary process of MRP systems, illustrated on figure 1., also meant, that MRP II systems took over the role to tie the basic MRP system to the company's *financial system* and other core processes (Krajewski et al. 2007). This enables us to calculate the exact costs to maintain a unit of capacity (a machine), including the cost of capital, the cost of labor force and other incidental expenses.

5. An integer programming model for capacity planning

Using the information listed above we can formulate a general linear programming model, which rationalizes capacity utilization. We build this model based on a paper presented by the author and two co-authors on an international conference (Naghi et al. 2009), and we also add some minor modifications. The following model aims to minimize the total operating cost of the production capacity used, while satisfying market demand (lost sales are not allowed).

For a general linear programming model we first need to determine the values of the two parameters listed below:

• *ct* – the average number of days of order cycle times

• *th* – the average number of days after which the company can make decisions regarding capacity dimensioning.

With the help of these to parameters, we can find out how many orders occur over one capacity planning horizon (*T*), which is the average number of days in a capacity planning horizon (*th*) divided by the order cycle time (*ct*):

$$T = th \,/\, tc$$

For example in the case study presented in this paper we use the following time periods:

• Average order cycle time is one week – after receiving the placed order from the customer the company has one week to produce the demanded quantity

• Capacity planning time horizon is one month – the company can make capacity resizing decisions on a monthly basis

Having this information, the objective function of the linear programming model can be formulated as follows:

$$\min z = \sum_{t=1}^{T} \left(\sum_{i=1}^{n} x_i \cdot c_i + \sum_{i=1}^{n} x_i \cdot to_i^{\,t} \cdot co_i \right) ,$$

where the elements of the objective function denote the following quantities:

z – objective function value that has to be minimized; *z* represents the total cost needed to operate the production capacities (machines) per month (both in regular time and overtime);

t – current index of order cycle time

T – number of order cycle times in one capacity planning time-horizon

i – index number denoting the current machine type

n – number of machine types used by the company

x_i – represents the vector of the decision variables (X), which contains the current number available of every i^{th} type production equipment needed in a month ($i = \overline{1, n}$):

$$X = [\ x_1, x_2, \dots, x_n\ ];$$

c_i – stands for the operating cost of the i^{th} machine type in regular time during one order cycle time. Here we include all of the expenses which would not become due in case of the machine was not used.

If the i^{th} machine type is rented or leased by the company over the planning horizon c_i can be calculated as follows:

$c_i =$ rent or lease payment per week +
cost of labor force needed to operate the machine per week +
average incidental costs per week (maintenance costs, repairing costs etc.).

In case the i^{th} machine type is not rented or leased, we assume that it was purchased by the company to be used for a certain period of time (useful lifetime of the machine). In this case the price of the machine can be converted into equivalent payments in every order cycle time, using an annuity formula, dispersing the investment costs over the lifetime of the machine. We can determine these payments by using the annuity formula, given below (Brealey - Myers 2005):

$$PV = C \cdot \left(\frac{1}{r} - \frac{1}{r \cdot (1 + r)^t} \right), \text{ where}$$

PV – present value of the annuity (C yearly payments), in our case the amount of the initial investment (the price of the machine)
C – yearly payments
r – average cost of capital used by the company for evaluating investment options
t – useful lifetime of the machine

Expressing the value of C from the formula above and dividing it by the number of order cycle times in a year (denoted by cty), we get the amount of weekly payment (c_i) equivalent with the initial purchase price of the machine:

$$c_i = \frac{1}{cty} \cdot \frac{PV}{\left(\frac{1}{r} - \frac{1}{r \cdot (1 + r)^t} \right)}.$$

The remaining elements of the objective function denote the following quantities:

$to_i^{\,t}$ – average number of hours worked in overtime by the i[th] machine type in period t. Overtime decisions during the capacity planning horizon can be represented by the following TO matrix ($t = \overline{1, T}$ $\;i = \overline{1, n}$):

$$TO = \begin{bmatrix} to_1^1 & \dots & to_n^1 \\ \vdots & \ddots & \vdots \\ to_1^T & \dots & to_n^T \end{bmatrix}$$

co_i – cost of operating the i[th] machine type in overtime per hour, including only the cost of labor and incidental expenses, excluding rent or lease payment.

We want to minimize the objective function, subject to the following constraints:

5.1. Monthly demand constraint

$$\sum_{j=1}^{m} D_j^t \cdot h_{ji} \le x_i \cdot (tr_i + to_i^t) \quad i = \overline{1, n};\; t = \overline{1, T}$$

m – number of product types manufactured by the company

$D_j^{\,t}$ – quantity demanded of the j[th] product type in period t. The forecasting module of the production planning and control system should provide the projection of demand for the following capacity planning period for each product type, which can be written in the following matrix (D) form ($t = \overline{1, T}$ $\;j = \overline{1, m}$):

$$D = \begin{bmatrix} D_1^1 & \dots & D_m^1 \\ \vdots & \ddots & \vdots \\ D_1^T & \dots & D_m^T \end{bmatrix}$$

h_{ji} – manufacturing hours needed on the i[th] machine to produce the j[th] product type ($j = \overline{1, m}$ $\;i = \overline{1, n}$)

$$H = \begin{bmatrix} h_{11} & \dots & h_{1n} \\ \vdots & \ddots & \vdots \\ h_{m1} & \dots & h_{mn} \end{bmatrix}$$

Every row of the matrix above represents the operations list for different product types, indicating how much operation time a certain product requires on different machines.

tr_i – total regular operating time available for machine i in a week, which remains constant during one capacity planning horizon ($i = \overline{1, n}$)

$$TR = [tr_1, tr_2, ..., tr_n]$$

Regular operating time per week can be calculated as:

tr_i = (number of working days in one order cycle period) ×

× (available working hours for machine i per day).

The demand constraint expresses that there has to be enough capacity during every period to fulfill the demand by producing in regular time and in overtime.

5.2. Overtime constraint

$$to_i^t \leq tr_i \cdot otl\ (\%) \quad i = \overline{1, n}\,;\, t = \overline{1, T}\,,\ \ where$$

otl(%) – the maximum limit of overtime per period, expressed as a percentage of regular operating time per period. The value of this maximum limit can be determined according to the regulations of the current legal system regarding labor force or can be found in a contractual framework with the workers or syndicate.

5.3. Non-negativity constraints

$$x_i \geq 0 \quad i = \overline{1, n}$$

$$to_i^t \geq 0 \quad i = \overline{1, n} \quad t = \overline{1, T}$$

The value of every decision variable should be greater than or equal to zero.

5.4. Integer constraint

$$x_i\ integer \qquad i = \overline{1, n}$$

The value of the x_i decision variable, representing the number of type i machines should have an integer value. By introducing this latter constraint we transform the linear programming problem into an integer programming model.

Summarizing the integer programming model built in this chapter we get:

Decision variables:

$X = [x_1, x_2, ..., x_n]$ – number of machines of different types

to_i^t - overtime decisions (hours) for each machine type and each period of the capacity planning horizon ($t = \overline{1, 4}$ $i = \overline{1, n}$):

$$TO = \begin{bmatrix} to_1^1 & \dots & to_n^1 \\ \vdots & \ddots & \vdots \\ to_1^T & \dots & to_n^T \end{bmatrix}$$

Parameters:

c_i – one period's operating cost for machine i in regular time

co_i – one period's operating cost for machine i in overtime

D_j^t – projected demand for product j in period t

h_{ji} – manufacturing time for product j on machine i

tr_i – total regular operating time available for machine i per period

$otl(\%)$ – maximum limit of overtime per period, expressed as a percentage of regular time available

t – current number of period within the capacity planning horizon

n – number of different machine types used by the firm

m – number of different product types produced by the company

Objective function:

$$\min z = \sum_{t=1}^{T} \left(\sum_{i=1}^{n} x_i \cdot c_i + \sum_{i=1}^{n} x_i \cdot to_i^t \cdot co_i\right) ,$$

Subject to:

$$\sum_{j=1}^{m} D_j^t \cdot h_{ji} \le x_i \cdot (tr_i + to_i^t) \quad i = \overline{1, n}; \; t = \overline{1, T}$$

$$to_i^t \le tr_i \cdot otl\,(\%) \quad i = \overline{1, n}; \; t = \overline{1, T}$$

$$x_i \ge 0 \quad i = \overline{1, n}$$

$$to_i^t \ge 0 \quad i = \overline{1, n}; t = \overline{1, T}$$

$$x_i \; integer$$

6. Case study

In the present case study we analyze some aspects of the capacity planning process at a Romanian small-sized enterprise from the textile industry, based on a paper presented on an international conference (Naghi et al. 2009). The firm executes textile operations on a make-to-order basis, working in „lohn" system. The „lohn" production system means that materials, parts, subassemblies and semi-finite products needed to execute the production process are delivered by the customer, who places the order. In such a system basic MRP tasks like material handling, inventory control and procurement are handled by the customer.

After only six months of operations the firm was forced to declare bankruptcy, due to the extremely high operating expenses. In the present case study we analyze if the firm – using the capacity planning model presented in the previous chapter – could have improved the efficiency of the production system by rationalizing capacities and decrease operating expenses.

6.1. Production resources

Looking at the material resources used by the firm these were specific to the textile and clothing industry. All types of material resources were delivered by the client, who was responsible for shipping the right quantity at the right time for the firm. The effect of the "lohn" system on the cost-structure of such a firm can be easily understood if we look at the percentage of material costs related to the total costs of the firm in one period. Using data from the monthly income statement of the firm we can calculate the average value of this ratio over the lifetime of the firm, which was equal to 0.545%, a percentage that can be considered insignificant.

Having such low material costs, machine operating costs and the cost of labor accounted for a major part of the firm's total expenses. Consequently, capacity decisions were a major determinant of the firm's overall performance. The firm was using 10 different types of machines (for simplicity let us denote them with capital letters: A, B, ... , J), each of them requiring one qualified worker. The machines were rented on a monthly basis (for a fee of 50 RON/equipment/month). The actual number of different machine types is presented in the table below:

Table 2. Number of different machine types used by the firm

	A	B	C	D	E	F	G	H	I	J
No. of machines	2	2	1	1	10	4	1	3	1	1

Source: own calculations

6.2. Parameters of the integer programming model

As capacities were rented on a monthly basis, the length of the planning horizon is set to one month, and average order cycle time was one week. Therefore, the value of T (number of weeks in a month) is set to 4.

The number of different machine types: $n = 10$.

The number if different product types produced by the company: $m = 6$. Further on, we will denote the different product types with *T1*, *T2*, *T3*, *T4*, *T5* and *T6*.

To construct the master production schedule (MPS) we use order entries from the month with the maximum demand. Having the order entries of this month we construct the MPS, indicating produced quantities for each week and each product type:

Table 3. Master production schedule for a month

MPS	T1	T2	T3	T4	T5	T6
Week 1	0	0	0	72	360	138
Week 2	0	383	0	107	0	0
Week 3	216	336	0	0	0	0
Week 4	312	0	297	0	0	0

Source: own calculations

Data from the table above represent the values of the *D* matrix, containing the produced quantities of every product *j* in week *t*.

To construct the operations list, which contains the duration and type of operations needed for every type of product, we use the technological descriptions of the production process for every product type. The table below contains manufacturing times in minutes for one unit of each product type on each machine. If a certain machine is not needed to produce a certain product type, manufacturing time is set to zero.

Data from the 4. table represent the values of the *H* matrix containing values of h_{ji} ($j = \overline{1, m}$ $i = \overline{1, n}$) – manufacturing time needed for product *j* (in columns) on machine *i* (in rows).

Total regular operating time in a week for each machine (tr_i), expressed in minutes is equal to:

Days in a week:	5	
Effective operating hours:	7	
Minutes/hour:	60	
	2100	min.

Table 4. Manufacturing times (minutes)

		Product type					
		T1	T2	T3	T4	T5	T6
Machine type	A	3.904	7.861	0.000	1.202	2.372	1.657
	B	4.075	0.301	0.869	1.109	1.365	1.109
	C	5.240	4.831	0.000	3.911	3.911	3.911
	D	0.329	0.329	0.000	0.329	0.329	0.329
	E	19.759	19.970	6.675	11.697	12.940	11.697
	F	0.000	3.857	1.779	1.039	2.519	2.960
	G	1.260	0.630	0.000	1.260	1.260	1.260
	H	5.155	10.313	7.962	9.024	10.819	10.619
	I	0.000	0.000	0.000	2.252	3.770	2.252
	J	0.000	0.000	0.000	2.162	2.131	2.162

Source: own calculations

Note that the effective operating hours are less than 8 hours per day, due to 30 minutes maintenance time and a 30 minute lunch-break. Regular operating time is equal to 2100 minutes for every type of machine:

$$tr_i = 2100 \text{ min} \quad i = \overline{1, n}$$

The maximum limit of overtime is set to 10% of regular operating time per week (*otl(%)=10%*), which is equal to a maximum of 3.5 hours overtime per week.

Calculating weekly operating costs for the different types of machines includes the cost of capital, cost of labor force needed to operate a machine and the cost of maintenance and breakdowns. Supposing that monthly rent is 50 RON/machine, monthly gross salary of one qualified worker is 900 RON and maintenance costs are 50 RON/machine, total operating cost per machine in regular time can be calculated as follows:

Rent per week:	12.5	+
Labor cost per week:	225	
Maintenance cost per week:	12.5	
	250	RON

In our case total weekly operating cost is the same for each type of machine. Therefore, regular operating costs are equal to 250 RON for every machine:

$$c_i = 250\ RON \quad i = \overline{1, n}$$

Overtime operating costs for each machine type will include only the cost of labor (which is 25% higher than in regular time) and the additional cost of maintenance. We calculate overtime operating cost per minute, since values of to_i^t – total overtime of one unit of machine *i* in week *t* are also expressed in minutes.

Labor cost per minute:	0.117 +
Maintenance cost per minute:	0.005
	0.122 RON

Note that labor cost per minute in overtime is 25% higher than in regular time. Overtime operating costs are constant for every machine type.

$$c_i = 0{,}122\ RON \quad i = 1, n$$

6.3. Building and solving the integer programming model

Having all the parameters calculated as seen above we can build an integer programming model to search for the optimal capacity planning decision at our firm.

Objective function:

$$\min z = \sum_{t=1}^{4} \left(\sum_{i=1}^{10} x_i \cdot 250 + \sum_{i=1}^{10} x_i \cdot to_i^t \cdot 0.122\right)$$

Subject to:

$$\sum_{j=1}^{6} D_j^t \cdot h_{ji} \le x_i \cdot (2100 + to_i^t) \quad i = \overline{1, 10};\ t = \overline{1, 4}$$

$$to_i^t \le 2100 \cdot 10\% \quad i = \overline{1, 10};\ t = \overline{1, 4}$$

$$x_i \ge 0 \quad i = \overline{1, 10},\ x_i\ integer$$

$$to_i^t \ge 0 \quad i = \overline{1, 10};\ t = \overline{1, 4}$$

Left-hand side parameters and the optimal right-hand side values (for each machine i and each week t) of the first constraint can be found in the Appendix.

By calculating the optimal solution to this linear programming model (for example with Microsoft Excel's Solver function) we get the optimal values of the decision variables (number of machines, quantity of overtime used) at which the value of total operating cost per month (z) is minimal.

The table below contains the optimal integer values for each machine type to be used (x_i):

Table 5. Optimal number of different machine types

	A	B	C	D	E	F	G	H	I	J
No. of machines	2	1	2	1	5	1	1	3	1	1

Source: own calculations

This optimal solution requires an overtime of 95.57 minutes in week 3 only for machine E, which means that:

$$to_5^3 = 95.57 \text{ , and}$$

$$to_i^t = 0 \quad i = \overline{1, 10}\ ; t = \overline{1, 4} \text{ where } i \neq 5 \text{ and } t \neq 3$$

Using the optimal values of the decision variables we calculate the optimal value of the objective function:

$$z = 4558.49 \ RON$$

Hence, total operating cost of capacities in a month (including regular and overtime) is equal to 4558,49 RON, from which:

- Regular time cost = 4500 RON
- Overtime cost = 58.49 RON

To evaluate what improvements this model could have brought to the firm, we calculate the total operating cost per month based on the initial capacity configuration (see *Table 2*) in which case the z value equals 6500 RON. This value is much higher then the optimal objective function value, meaning that an almost 30% cost decrease would have been achievable (-29.87%). It is not sure if this cost reduction could have been the most critical factor to avoid bankruptcy, but it substantially improves the efficiency of the production system, increasing the competitiveness of the firm.

The model can also be used to make optimal decisions about the timing of capacity expansions or reductions. By running this linear programming model for several consecutive months the decision maker can follow the optimal level of capacities for each period and decide about expanding it or – as in our case – reducing it.

7. Conclusions

In the present paper we reviewed the main challenges of the capacity planning process and argued that capacity decisions can greatly influence the overall performance of a firm. We analyzed the structure and hierarchy of capacity planning decision and identified information (input data) needed to perform the capacity planning process.

Based on the information provided by an MRP/MRP II based production planning and control system we developed a linear programming model with integer variables, which determines the optimal dimension of production capacities by minimizing total operating costs. As demonstrated in the case study, this model can be used to bring substantial improvements to the efficiency of a production system and can also help decision makers to make optimal decisions about capacity expansions or reductions at the right moment of time. In this latter case the integer linear programming model has to be run using as input data the master production schedules of several future time periods. The model indicates the optimal capacity levels for each period; however, it does not determine the optimal capacity dimension for a longer period of time.

Still, one of the model's most important limitations is that it executes capacity planning only on an aggregate level, by making a rough-cut capacity check based on the master production schedule. The model does not take into account other parameters from a lower level of aggregation, like lot sizing rules, setup times or bottleneck capacities. However, it takes into consideration the possibility of a firm to use overtime production. The model can be further refined by adding other constraints, reaching down to lower levels of aggregation and including other special costs like setup costs.

Appendix

1. The matrix of the necessary machine times for each machine type in each week to satisfy market demand (Left-hand side values of the first constraint, L):

Machine	A	B	C	D	E	F	G	H	I	J
Week 1	1169.1	724.3	2229.3	187.5	7114.8	390.1	718.2	6010.0	1830.1	1221.2
Week 2	3139.4	233.9	2268.8	161.2	8900.1	1588.4	376.1	4915.4	241.0	231.3
Week 3	3484.6	981.3	2755.1	181.6	10977.9	1296.0	483.8	4578.6	0.0	0.0
Week 4	1218.0	1529.5	1634.9	102.6	8147.3	528.4	393.1	3973.1	0.0	0.0

Every element of the matrix above is calculated as follows:

$$L_{ti} = \sum_{j=1}^{6} D_j^t \cdot h_{ji} \quad i = \overline{1,\ 10};\ \ t = \overline{1,\ 4}$$

2. Available machine times per machine type in each week, in case of optimal capacity configuration - assuming optimal number of different machine types (see Table 5)

(Right-hand side of the first constraint, R):

Machine	A	B	C	D	E	F	G	H	I	J
Week 1	4200	2100	4200	2100	10500	2100	2100	6300	2100	2100
Week 2	4200	2100	4200	2100	10500	2100	2100	6300	2100	2100
Week 3	4200	2100	4200	2100	10977.9	2100	2100	6300	2100	2100
Week 4	4200	2100	4200	2100	10500	2100	2100	6300	2100	2100

Every element of the matrix above is calculated as follows:

$$R_{ti} = x_i \cdot (2100 + to_i^t) \quad i = \overline{1,\ 10};\ \ t = \overline{1,\ 4}$$

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